Getting More from Less

Independent Simultaneous Self-Restraint Games

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Thesis Overview

1. Independent Simultaneous Self-Restraint (ISSR) Games (Today)
2. Application of ISSR games to airline pricing (In process)
   - Descriptive evidence on airline pricing
   - Structural model of airline prices/fare availability

Example

	p_2^H	p_2^M	p_2^L
p_1^H	$(15, 15)$	$(5, 20)$	$(1, 11)$
p_1^M	$(20, 5)$	$(10, 10)$	$(6, 8)$
p_1^L	$(11, 1)$	$(8, 6)$	$(5, 5)$

Intuition: How to Get More from Less

To get a **reward** (p_i^H), players choose a set of **punishments** (p_i^L) to motivate other players to get rid of their **temptations** (p_i^M).

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Four important assumptions

1. No restrictions on A_i
2. Ability to commit to A_i
3. Simultaneity of moves
4. Public knowledge of A_i

Related literature

- Strategic players may benefit from reduced flexibility (e.g., Stackelberg (1934) and Schelling (1960) by moving first)
  - Reduced flexibility is one-sided, commitment to a single action
- Contracts may serve as a mutual commitment device not to play certain strategies (e.g., Hart and Moore (2004) and Bernheim and Whinston (1998))
  - Mutual cooperation is explicit and facilitated by contracts
- Cooperation can be supported in repeated games when players can sufficiently punish deviators (e.g., Abreu, Pearce, and Stacchetti (1990) and Fudenberg and Maskin (1986), among others)
  - Cooperation is supported by repeated interactions of sufficiently patient players
- Bilateral commitment to convex subsets (Bade, Haeringer, and Renou (2009))
  - Limited ability to facilitate cooperation

Overview of the talk

- Formal Setup
- ISSR and NE outcomes
- Two results for subgame supermodular games

Game G: one-shot normal form game

- normal-form game $G = (\mathcal{I}, \mathcal{A}, \pi)$
- set of players: $\mathcal{I} = \{1, 2, ..., n\}$
- action spaces: $\mathcal{A}_i$
- payoff functions: $\pi_i : \mathcal{A}_1 \times \mathcal{A}_2 \times ... \times \mathcal{A}_n \longrightarrow \mathbb{R}$
- outcome: $a \in \mathcal{A}$
- payoff: $\pi(a) \in \mathbb{R}^n$
- solution concept: pure-strategy Nash equilibrium
- set of all NE outcomes: $\mathcal{E}_G$

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Assumptions

- $\mathcal{A}_i$ is a compact set of $\mathbb{R}$
- π_i is continuous in (a_i, a_{-i})

Game C(G): ISSR game

- two-stage game:

  1. Commitment stage**:** choose a non-empty *compact* subset: $A_i \in \mathbb{A}_i \subseteq 2^{\mathcal{A}_i} \setminus \{\varnothing\}$. A_i is publicly observed.

  2. Action stage: each player simultaneously and independently chooses an action $a_i \in A_i$. Actions not in A_i are not permitted.

- solution concept: subgame perfect pure-strategy Nash equilibrium (called ISSR equilibrium)
- set of all ISSR eqm outcomes: $\mathcal{E}_C$

ISSR and NE

Theorem

$\mathcal{E}_G \subseteq \mathcal{E}_C$.

ISSR and NE

Theorem

$\mathcal{E}_G \subseteq \mathcal{E}_C$.

Theorem

(i) If $\mathbb{A}_i = \{A_i : A_i \equiv \mathcal{A}_i\}$*, then* $\mathcal{E}_G = \mathcal{E}_C$.
(ii) If $\mathbb{A}_i = \{A_i : |A_i| = 1\}$*, then* $\mathcal{E}_G = \mathcal{E}_C$.

Thus, to get an outcome outside $\mathcal{E}_G$:

- players have to constrain their action sets
- players have to choose more than one action

Overview of the talk

- Setup
- ISSR and NE outcomes
- Two results for subgame supermodular games
    - what it is?
    - why we need them?
    - what we can achieve there?

Subgame supermodular games

Definitions

(i) Game $C(G)$ is called *subgame supermodular* if any subgame is supermodular.
(ii) Game G is called *supermodular* if for every player i, π_i has increasing differences in (a_i, a_{-i}).

Lemma

$C(G)$ is subgame supermodular if and only if G is supermodular.

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Lemma

$C(G)$ is subgame supermodular if and only if G is supermodular.

Theorem

If $C(G)$ is subgame supermodular, then an ISSR equilibrium exists.

In other games, there could exist a subgame induced by a unilateral deviation without pure-strategy eqm

Theorem

Suppose that π_i is strictly quasi-concave in a_i for both i. Suppose $a^* = (a_1^*, a_2^*) \in A_1 \times A_2$ can be supported by an ISSR equilibrium. Then
(i) if $a_i^* < BR_i\left(a_{-i}^* \,|\mathcal{A}_i\right)$, then $a_{-i}^* \leq BR_{-i}\left(a_i^* \,|\mathcal{A}_{-i}\right)$;
(ii) if $a_i^* > BR_i\left(a_{-i}^* \,|\mathcal{A}_i\right)$, then $a_{-i}^* \geq BR_{-i}\left(a_i^* \,|\mathcal{A}_{-i}\right)$,
where $BR_i\left(a_{-i}|A_i\right) = Arg\max_{a_i \in A_i} \pi_i\left(a_i, a_{-i}\right)$.

Intuition: To support an ISSR eqm, a player's incentives to deviate should coincide with other players' incentives to punish. Otherwise, profitable deviations exist.

Relative to a NE outcome	Bertrand	Cournot
Pareto superior outcome (reward)	higher p	lower q
Temptations	lower p	higher q
BR to a temptation	decrease p	decrease q
Effective punishment	lower p	higher q
Punishment is	credible	NOT credible

Thus, firms cannot get more from less in Cournot games.

Definition

An outcome $\left(a_i^L, a_{-i}^F\right)$ is called *a Stackelberg outcome for player* i, if (i) $a_i^F \in BR_{-i}\left(a_i^L \,|\mathcal{A}_{-i}\right)$ and (ii) $\pi_i\left(a_i^L, a_i^F\right) \geq \pi_i\left(a_i, a_{-i}\right)$ for any $a_i \in \mathcal{A}_i$ and $a_{-i} \in BR_{-i}\left(a_i \,|\mathcal{A}_{-i}\right)$.

Definition

$L = \left\{a \in \mathcal{A}\colon\ \pi_i\left(a\right) \geq \pi_i^L \text{ for both } i\ \right\}$, where $\pi_i^L = \pi_i\left(a_i^L, a_{-i}^F\right)$

Lemma

L is not empty.

Theorem

Suppose π_i is strictly quasi-concave in a_i and increasing in a_{-i}. Then an outcome $a^* \in L$ can be supported by an ISSR equilibrium if and only if for both i there exists an $a_i^P \neq a_{-i}^*$ satisfying $\pi_i\left(a_i^P, a_{-i}^*\right) = \pi_i\left(a_i^*, a_{-i}^*\right)$.

Intuition: In an ISSR equilibrium, players have to be indifferent between playing their reward and punishment actions. Otherwise, small deviations are profitable.

Example

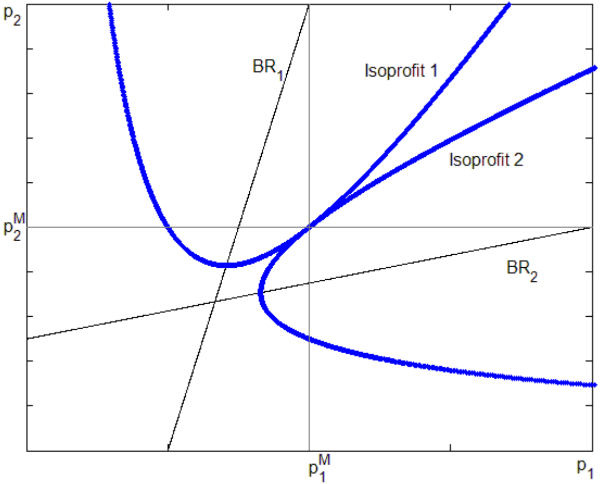
- two firms produce differentiated products
- linear demand systems

$$
\begin{aligned}
q_1(p_1, p_2) &= 1 - p_1 + \alpha p_2 \\
q_2(p_1, p_2) &= 1 - p_2 + \alpha p_1 \\
\alpha &\in (0, 1)
\end{aligned}
$$

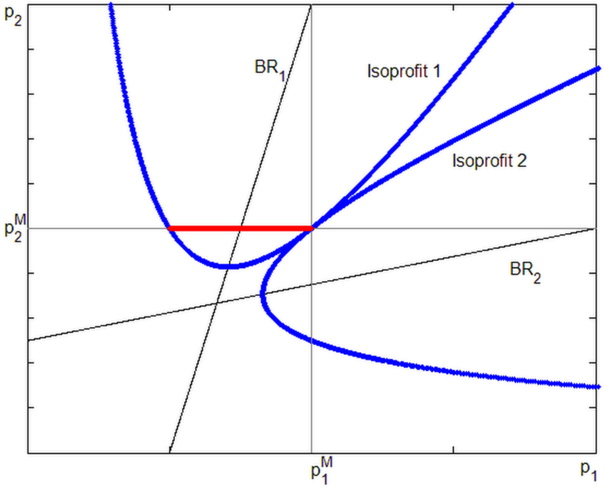
- costs are normalized to zero

Example

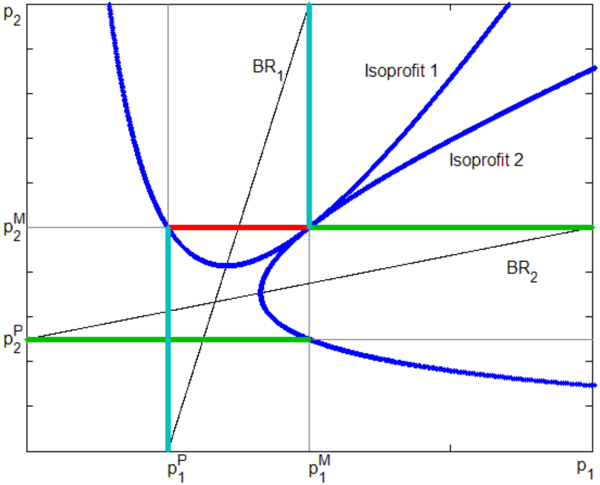
Joint-profit maximization

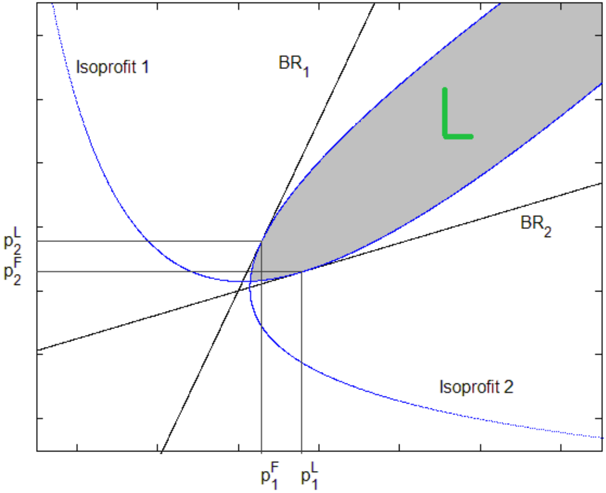


Example
Temptations



Example
Punishment





Isoprofit 1
BR1
L
BR2
Isoprofit 2
p_2^L
p_2^F
p_1^F
p_1^L

Summary of other results in the paper:

- For continuous games:
  - ISSR equilibria can support outcomes that are Pareto inferior to NE
  - ISSR equilibria may not be able to support mixed strategy NE
  - ISSR equilibria may exist even if G has no pure-strategy NE
  - Properties of BR: Indifference principle
- For supermodular games:
  - It is sufficient to prevent deviations to singleton subsets
  - ISSR equilibria with one punishment: a necessary and sufficient condition
  - ISSR equilibria: when having more punishments doesn't help
  - A partial characterization of the set of equilibrium payoffs

Extensions

- N Players
  - A punishment of $N-1$ players is more severe than a punishment of one player.
  - If one player punishes, the others have more incentives to punish (e.g. price war).
  - Different punishments may be used for punishing different players. In asymmetric equilibria, different players may use different punishments.
  - There could be multiple sets of punishments that will support the same outcome.
- Stochastic payoffs
  - Being first or being right: commitment vs. flexibility
- Multiple stages
  - Multiple repetition of commitment and/or action stage